|  |
| --- |
|  |

***Reduction formula***: A reduction formula is a formula which connects a given integral with another integral which is of the same type but of a lower degree or of a lower order, or is otherwise easier to evaluate.

**Working rule to find a reduction formula for :**

1. Separate  from . Thus .
2. Replace  by .
3. Integrate only the first integral on R.H.S. using .

**Problem-01**: Find a reduction formula of the integral  and hence find .

Given that













which is the required reduction formula.

**2nd part:** We have



Putting  in (1), we get







 ***Ans.***

**Problem-02**: If  then show that  and hence find the value of

.

Given that



















which is the required reduction formula.

**2nd part:** We have



Putting  in (1), we get

















 ***Ans.***

**Working rule to find a reduction formula for :**

1. Separate  from . Thus .
2. Integrate by parts taking  as first function.
3. Replace  by .
4. Transpose the given integral to L.H.S.

**Problem-03**: Establish the reduction formula for  and hence find .

Given that























which is the required reduction formula.

**2nd part:** We have



Putting  in (1), we get









 ***Ans.***

**Working rule to find a reduction formula for :**

1. Separate  from . Thus .
2. Integrate by parts taking  as first function.
3. Replace  by .
4. Transpose the given integral to L.H.S.

The working rule to find a reduction formula for  is same as above.

**Problem-04**: Establish the reduction formula for  and hence find .

Given that























which is the required reduction formula.

**2nd part:** We have



Putting  in (1), we get



















 ***Ans.***

**Working rule to find a reduction formula for ** or **:** Integrate twice by parts taking  as first function.

**Problem-05**: Establish the reduction formula for  and hence find .

Given that















which is the required reduction formula.

**2nd part:** We have



Putting  in (1), we get















 ***Ans.***

**Working rule to find a reduction formula for ** or **:**

1. Write as  or as .
2. Integrate by parts taking  or as first function and replace  by  and  by .
3. Transpose and solve for the given integral.

**Problem-06**: If  then prove that , . Hence prove that

.

Given that





















 **(Proved)**

which is the required reduction formula.

**2nd part:** We have



Putting  in (1), we get













. **(Proved)**

**Working rule to find a reduction formula for**  **:**

**Problem-07**: Find the reduction formula for .

Given that

























which is the required reduction formula.

**Problem-08**: If  , then show that .

Given that

























 **(Showed)**

**Working rule to find a reduction formula for** or  oror  **:**

**Problem-09**: Find the reduction formula for  and hence find the value of

.

Given that























which is the required reduction formula.

**2nd part:** We have













 ***Ans.***

**Problem-10**: Find the reduction formula for  and deduce the value of

.

Given that





















which is the required reduction formula.

**2nd part:** We have





















 ***Ans.***

**Problem-11**: Obtain a reduction formula for  and hence evaluate.

Given that











Replacing *n* by  in (1), we get



Adding (1) and (2), we get

















which is the required reduction formula.

**2nd part:** We have



Putting  in (3), we get



 





 ***Ans.***

**Problem-12**: Obtain a reduction formula for ,and hence evaluate.

Given that





























which is the required reduction formula.

**2nd part:** We have



Putting in (1), we get











 ***Ans.***

**Problem-13**: Obtain a reduction formula for .

Given that









which is the required reduction formula.

**Problem-14**: Obtain a reduction formula for .

Given that





















Replacing *n* by , we get







which is the required reduction formula.

**Theorem-01:** State and prove Wallis’s formula.

**OR**

Evaluate  and  for all positive odd and even integral values of .

**Statement:** If  is positive integer, then



**Proof:** Let 



































, when *n* is even









Again,











, when *n* is odd







Thus  **(Proved)**

***Assignment:***

**Problem-01**: If  then show that  and hence find the value of

.

**Problem-02**: If  then show that  and hence find the value of

.

**Problem-03**: Establish the reduction formula for  and hence find .

**Problem-04**: If  then show that  and hence find the

value of .

**Problem-05**: If  then for show that .

**Problem-06**: Find the reduction formula for .

**Problem-07**: If  then show that .

**Problem-08**: Find the reduction formula for .

**Problem-09**: If  , then show that  and hence find , when *n* is odd.

**Problem-10**: If *n* is a positive integer, then prove that.

**Problem-11**: If , then show that .

**Problem-12**: If , then show that .

**Problem-13**: If , then show that .